

**Improving Assimilative Capacity Modelling for Scottish
Coastal Waters: II. A Model of Physical Exchange for Open
Water Sites**

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Introduction

This report describes progress towards achieving the second milestone of the project entitled “The development of modelling techniques to improve predictions of assimilative capacity of water bodies utilised for marine caged fish farming” which was funded by the Scottish Aquaculture Research Forum (Project code SARF012). The second milestone is to consider, and develop a model of, the physical exchange of water at open coastal aquaculture sites. In Scotland, the majority of salmon production sites are located in sheltered, semi-enclosed sea lochs and voes (Regions of Restricted Exchange, RRE); modelling the exchange of RREs has been undertaken in Objective 1. A sizeable minority of salmon farms in Scotland are found in more open coastal waters, either in straits and channels (e.g. the Sound of Mull) or in relatively exposed water bounded on one side only by the coastline. The open, exposed nature of these sites, and the lack of a clearly defined boundary surrounding a fixed body of water (as in an RRE), means that the RRE model is not applicable to these sites, and a different, more flexible, model is needed to predict the exchange and assimilative capacity of the water body in which the farm is located.

In this report, we describe the conceptual framework for an open water model, describe the equations that govern the exchange, and explain the implementation of the model and the output that is provided. Some test case results are presented and implications regarding the influence of horizontal eddy diffusion discussed.

The model code is written in Matlab[®] and is provided as scripts.

Description of the Model

Conceptual Framework

In response to the European Union Urban Waste Water Directive, the UK Comprehensive Studies Task Team (CSST, 1997) provided guidelines covering several aspects of effluent dispersion in coastal waters. Among the guidelines, the CSTT recommended that impacts due to nutrient discharges should be considered on a range of spatial scales in which the impacts may be different: Zone A is a region very close to the nutrient source; Zone B is the near-field region, typically of the same spatial scale as a tidal excursion; Zone C is the far-field region, where residence time is of the order of weeks to months. In Zone B, residence times are of the order of days and phytoplankton may grow within the zone if conditions are favourable. The model developed here considers the exchange of water in the Zone B scale region surrounding coastal aquaculture sites. (Note that the effluent considered here is dissolved inorganic nutrients, and we use the terms ‘effluent’ and ‘nutrients’ interchangeably).

CSTT (1997) considered that the size and exchange of Zone B is driven by tidal dynamics and diffusion. However, Sherwin (2001) pointed out that, in fact, the exchange is often dominated by the residual flow. This flow can result from rectification of tidal currents, wind forcing, or density gradients (deriving from river discharges), and can be derived from current meter observations. We will

follow Sherwin (2001) in defining the size of the Zone B area using the tidal excursion, and calculating the exchange rate from a combination of the residual flow and horizontal diffusion.

A sketch of Zone B around a nutrient source (e.g. a salmon farm) is presented in Figure 1. Zone B represents the volume of water into which nutrients released from the source are rapidly mixed (within a tidal cycle). If the nutrient release is continuous, as with a salmon farm, then the nutrient loading into Zone B is continuous, and the concentration in the body of water depends on the rate at which it is replaced (i.e. exchanged) by uncontaminated water. The calculation of the Zone B volume and its exchange rate are described in the next section.

Some assumptions are necessary. First, we assume that the flow at the site contains a tidal component and that the dominant tide is the semi-diurnal lunar (M_2) constituent. This will generally hold true throughout most UK coastal waters. Second, we assume that the vertical extent of the contaminant (nutrient) plume is bounded either by the seabed or by a pycnocline (at a depth specified by the user) and that the contaminant mixes evenly vertically through this surface layer.

Governing Equations

From Figure 1, we see that the size of zone B is determined by its length, L , and width, W , which are based on the tidal excursion i.e.

$$L = \frac{UT}{\pi} \quad \text{and} \quad W = \frac{VT}{\pi} \quad (1)$$

where U is the amplitude of the semi-diurnal tidal current (m s^{-1}), T is the semi-diurnal tidal period (s) and $\pi = 3.1415$. We assume, for simplicity, that the zone B volume, V , is rectangular and given by

$$V = LWH \quad (2)$$

where H is the water depth or pycnocline depth (m) and is specified by the user. It is assumed that nutrient (or other effluent) from the source is distributed uniformly within the zone.

The concentration of effluent within the box is determined by the exchange rate, E , which is the inverse of the flushing time, τ . The flushing time is defined as the time after which the mean concentration, \bar{C} , in the box would have fallen to a value of $C = C_0 e^{-1}$ ($=0.37C_0$) due to the action of physical exchange processes (advection and diffusion) only, where C_0 is the initial concentration. The exchange rate $E = 1/\tau$. If the exchange results from a number of different processes, then the individual exchange rates can be summed to provide a total exchange rate.

In the present model, following Sherwin (2001), we define the exchange rate E as:

$$E = E_A + E_X + E_Y \quad (3)$$

where E_A is the exchange rate due to advection (the residual flow), and E_X and E_Y are due to along-shore and across-shore diffusion respectively.

Each of these individual terms can be calculated simply. The exchange rate due to advection is given by

$$E_A = \frac{Q_R}{V} = \frac{U_R}{L} \quad (4)$$

where U_R is the residual flow and $Q_R = U_R \times W \times H$ is the volume flux due to the residual flow.

The exchange rates due to diffusion are given by

$$E_X = \frac{K_X}{2L^2} \quad (5a)$$

and

$$E_Y = \frac{K_Y}{2W^2} \quad (5b)$$

where K_X and K_Y are the along- and across-shore diffusion coefficients respectively. Derivations of these equations can be found in Sherwin (2001).

The exchange rates can then be used to calculate an effluent concentration, C , in zone B which is given by

$$C = \frac{S}{V(E_A + E_X + E_Y + k)} \quad (6)$$

where S is the effluent source (kg s^{-1}) and k is a decay rate (s^{-1}). Where the effluent is dissolved nutrients, C is the elevation of the nutrient concentration above background levels.

Equations 1-6 form the mathematical basis for the physical exchange model. For input values (or time series) of u , v , U_R , H , S and k , the model provides values (or time series) of V , E and C . In the next section, we describe how the input values can be provided to the model by the user.

Model Implementation and Data Provision

We envisage that the model will be driven by data collected in accordance with the requirements for Environmental Impact Assessments (EIAs). Typically, finfish producers are required to gather current meter data over a 15-day period to capture a spring-neap tidal cycle, over which most of the expected variance in current velocity will occur. It is assumed here that sites are subject to periodic tidal currents, and that the M_2 constituent with a period of 12.42 hours dominates the tides. However, calculating the amplitude of tidal currents, which are needed to

specify the size of zone B, requires specialist knowledge and software. We have therefore adopted a simpler method to estimate L and W, which uses the standard deviation of the east and north components of current velocity obtained from the 15-day observations as follows.

If the tidal current velocity, u , at time t is assumed to be sinusoidal in form and defined by

$$u = U \sin(\omega t) \quad (7)$$

where $\omega = 2\pi/T$ is the frequency of the semi-diurnal (M_2) tide, then it can be shown that the standard deviation, σ , of the velocity component u is given by

$$\sigma = \frac{U}{\sqrt{2}} \quad \text{or} \quad U = \sqrt{2}\sigma \quad (8)$$

Substituting (8) into (1) gives

$$L = \frac{\sqrt{2}\sigma T}{\pi} \quad (9)$$

This allows us to specify the size of the Zone B region based on the standard deviation of the current velocity data, an easier parameter to derive. If the current velocity data are resolved into along-shore and across-shore components, u and v , respectively, then the corresponding standard deviations, σ_u and σ_v respectively, can be used to define L and W i.e.

$$L = \frac{\sqrt{2}\sigma_u T}{\pi} \quad \text{and} \quad W = \frac{\sqrt{2}\sigma_v T}{\pi} \quad (10)$$

By providing a time series of current velocity data, therefore, values of σ_u and σ_v can be estimated, as can the residual velocity U_R . The user also needs to provide values of S, k as described below.

Relatively few data exist on the values of horizontal eddy diffusion in inshore waters of the North-west European continental shelf. Elliott (1997) suggested that $K_X \approx u \text{ m}^2 \text{ s}^{-1}$, where u is the current speed in m s^{-1} , and $K_Y \approx u/10 \text{ m}^2 \text{ s}^{-1}$. This suggests values of typically $K_X = 1 \text{ m}^2 \text{ s}^{-1}$ and $K_Y = 0.1 \text{ m}^2 \text{ s}^{-1}$. However, given that the diffusion coefficients are uncertain, we have chosen to make repeat calculations of the exchange rates and concentrations based on a range of along- and across-shore diffusion coefficients. The values chosen are:

$$K_X = [0.1, 0.2, \dots 1.0, 2.0, \dots 10.0] \text{ m}^2 \text{ s}^{-1}$$

$$K_Y = [0.01, 0.02, \dots 0.1, 0.2, \dots 1.0] \text{ m}^2 \text{ s}^{-1}$$

Calculations of E and C are performed for every combination of K_X and K_Y , a total of 361 ($= 19^2$) calculations being made. The output from the model includes the minimum, median and maximum values of E and C that result.

Mode of Operation

The model can be operated in two modes:

1. Single value
2. Time series

In the first mode, the user provides single values of all key parameters i.e. \bar{u} , \bar{v} , U_R , S and k . The single values of \bar{u} , \bar{v} and U_R might be derived from, and summarise, a 15-day time series of current velocity observations. In this mode, the model outputs single values of V , E and C , with minimum and maximum extremes of E and C also provided based on the range of horizontal diffusion coefficients.

In the second mode, time series of along-shore and across-shore velocity components, u and v , and nutrient source, S , are provided. The model derives estimates of \bar{u} , \bar{v} and U_R for every day (i.e. 24 hours) in the time series. Each daily analysis is performed on the subsequent 25 hours of data (i.e. the 24 hours of the day + the first hour of data from the next day). The 25-hour period is chosen so that when the residual velocity U_R is calculated (as the arithmetic mean of the 25-hour period of along-shore data), the M_2 tide, with a period of ca.12.5 hours, is averaged out. The standard deviations, \bar{u} and \bar{v} , are derived from the same 25-hour period. The user is required to input daily values of the nutrient source rate, S , for the duration of the time series. A single value of the decay rate, k , is also required.

In this mode, the model outputs time series of daily values of V , E and C . Values of E and C are the minimum, median and maximum values stemming from the range of diffusion coefficients K_X and K_Y . Results are output to a comma-separated ASCII file.

Results

Two examples of the model output are presented, one for each mode of operation.

Mode 1 'Single Value'

Figure 2 presents a screen dump during the application of the model, which shows the input and output of the model run for site 'test'. The zero passed to the model OWExR indicates that mode 1, 'Single value', is required.

The data passed to the model are as follows:

$$U_R = 0.05 \text{ m s}^{-1}.$$

$$\bar{u} = 0.75 \text{ m s}^{-1}.$$

$$\bar{v} = 0.075 \text{ m s}^{-1}.$$

$$H = 10 \text{ m}.$$

$$S = 100 \text{ kg day}^{-1}.$$

$$k = 0 \text{ s}^{-1}.$$

The resulting output is also shown in Figure 2. The Zone B volume is expressed in cubic metres and equates to a horizontal area of 23 km^2 (approx. $15 \text{ km} \times 1.5 \text{ km}$). The minimum, median and maximum values of the exchange rate, E , and the effluent concentration, C , are also given in SI units. The flushing time ($\tau = 1/E$) is calculated and presented in hours (note that the *minimum* flushing time is related to the *maximum* exchange rate). The effluent concentration, presented as an Equilibrium Concentration Enhancement (ECE) above background levels, is also presented in units of $\mu\text{mol l}^{-1}$.

Mode 2 'Time series'

For this example, we have used a 15-day time series of along-shore and across-shore velocity (Figure 3) to derive daily time series of \bar{u} , \bar{v} and U_R , as described above. We used values of S appropriate for a 1000-tonne salmon farm, derived from Davies (2000), who provided monthly values of nutrient emissions. To create our daily time series (Figure 4), the value of S for each day of every month was specified as the appropriate monthly value i.e. for every day in January, $S = 67 \text{ kgN day}^{-1}$, for every day in February, $S = 58 \text{ kgN day}^{-1}$ etc.

Since the time series of nutrient source was 365 days long, and the velocity data only 15 days long, we matched the two by repeating the velocity time series until a 365-day long record was created (Figure 5). The effect of this is that the velocity parameters, and therefore the exchange rate and flushing time, during the year have 15-day periodicities; since the nutrient source varies over the year, the effluent concentration has no such periodicity in its magnitude, although the pattern of C does repeat.

We have not utilised the decay rate here ($k = 0$), but the facility is available if required. The mixing depth, H , is set to 10m; in practise this will be determined by the water depth, or by data (profiles of temperature and salinity) showing the presence of a pycnocline. H can also vary daily if sufficient data are available.

The time series of velocity (u , v) and mixed layer depth (H) can be input to the model as Excel worksheets or as comma-separated ASCII text files.

Output from the model is shown in Figures 6 and 7. The time series of Zone B volume exhibits the 15-day periodicity discussed above and indeed, the size of the zone would be expected to vary with the changing current speeds over the spring-

neap cycle. Since both along- and across-shore current velocities at neap tides are about 50% of the values at spring tides, the Zone B volume varies by a factor of about 4 over the 15-day cycle.

The same periodicity is also evident in the time series of exchange rate, E , and, to a lesser extent for the effluent concentration. The changing magnitude of the effluent source, S , introduces a seasonal cycle into the concentration time series, with emissions peaking in the summer months.

In Figure 7, the time series of daily median values of E and C are plotted as the solid black line. The grey shading above and below indicates the range of values of E and C that result from the range of diffusion coefficients, K_X and K_Y , used in the calculations. For the velocity time series shown in Figures 3, with the resulting values of u , v and U_R (Figure 5), the exchange rate exhibits a fairly narrow range between minimum and maximum values. The daily residual velocity varies from about 1 cm s^{-1} to 10 cm s^{-1} , and the flow appears to dominate the exchange rate.

The effect of the uncertainty in the exchange rate is amplified in the time series of effluent concentration. Here, the minimum daily values can be as little as 25% of the median value, and the maximum as much as 75% greater than the median. It is clear then that uncertainty in the magnitude of horizontal diffusion can have significant implications for predictions of nutrient concentrations around fish farms. For the foreseeable future, given the poor state of knowledge on the relationship between horizontal eddy diffusion and the ambient flow field, the only solution is to make direct measurements of eddy diffusion at the site in question.

The model now needs to be tested against available observations, and this will be attempted under Objective 5 of the current project SARF012. That task, however, is fraught with difficulties. Although a large database of current meter observations now exists as a result of the EIA process, measuring the actual exchange of a poorly-defined volume of water has rarely been attempted (if at all). Using nutrient concentrations as a tracer, and comparing with model predictions, is also fraught with problems, particularly given that nutrients are not conservative and their uptake by plankton and bacteria in the water column is still poorly understood. It is important, therefore, to realise that the current model should be used as a screening model, and used to highlight areas where nitrification and eutrophication problems *may* arise in future, and to help specify an appropriate level of monitoring at the site.

Summary and Conclusions

A model to predict the size, exchange rate and nutrient concentrations of the Zone B region around aquaculture sites has been developed. The model runs in Matlab[®], and can be driven by single values, or time series, of velocity and nutrient source parameters, which can be provided as Excel spreadsheets or comma-separated ASCII files. The model deals with uncertainty in the magnitude of horizontal eddy diffusion in coastal waters by applying a range of realistic

along- and across-shore diffusion coefficients and calculating the minimum, median and maximum exchange rates and nutrient concentrations that result.

In many cases in the UK coastal zone, it is likely that the residual advective flow will dominate the exchange of the Zone B region. However, even small uncertainty in the exchange rate can have significant implications for predicted concentrations in the zone. The influence of the horizontal eddy diffusion is therefore important and for best, most reliable predictions of assimilative capacity at open water sites, direct measurements of the horizontal eddy diffusion should be made.

References

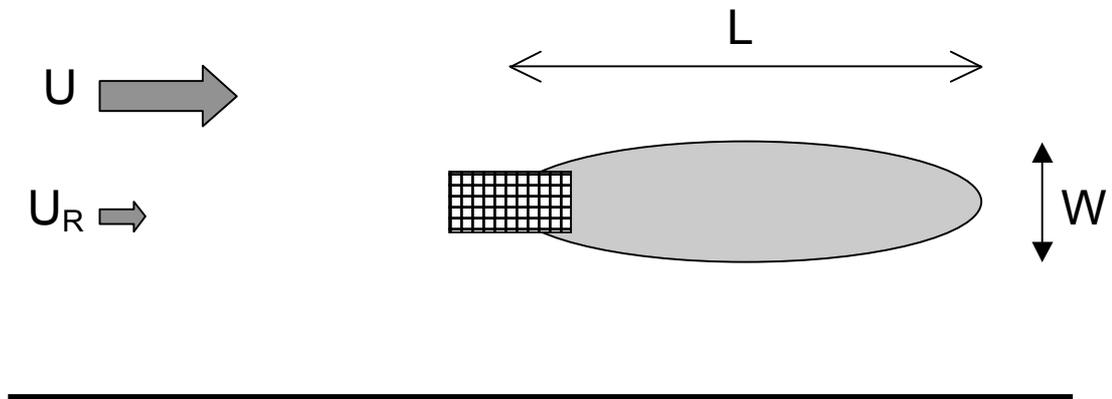
CSST (1997). Comprehensive studies for the purposes of Articles 6 & 8.5 of Directive 91/271 EEC, The Urban Waste Water Treatment Directive, 2nd ed. Marine Pollution Monitoring Management Group, Comprehensive Studies Task Team. Dept. of the Environment, Northern Ireland, Environment Agency, Scottish Environment Protection Agency and Water Services Association, 13 January 1997, pp. 51 + figs.

Davies, I.M. (2000). Waste production by farmed Atlantic salmon (*Salmo Salar*) in Scotland. ICES CM 2000/O:01, 13pp.

Elliott, A.J., Barr, A.G., Kennan, D. (1997). Diffusion in Irish coastal waters. Estuarine, Coastal and Shelf Science, 44 (Supplement A), 15-23.

Sherwin, T.J. (2000). The significance of residual currents in the interpretation of the EU Urban Wastewater Treatment Directive in coastal locations. Marine Pollution Bulletin, 40, 17-21.

(a) Flood tide



(b) Ebb tide

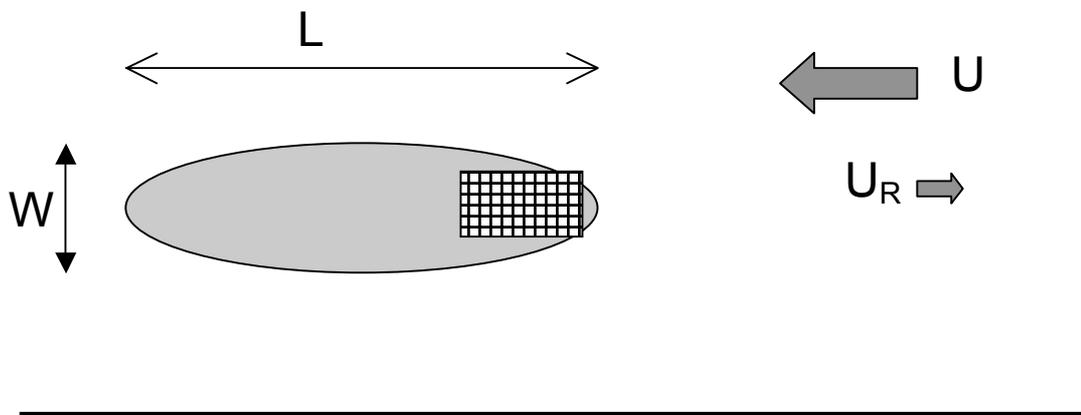


Figure 1. Simple sketch showing the plume of effluent (nutrient) discharging from an aquaculture site (hash-filled rectangle) during (a) flood tide, and (b) ebb tide. The tidal current amplitude (U) and the residual flow (U_R) are indicated. The length (L) and width (W) of the plume are indicated and defined in the text.

```
>>[V, E, C] = OWExR('test',0);
```

Site: test

Enter mean "along-shore" flow (m/s) : 0.05

Enter standard deviation of "along-shore" flow (m/s) : 0.75

Enter standard deviation of "across-shore" flow (m/s) : 0.075

Enter water depth or mixed layer depth (m) : 10

Enter nutrient source rate (kg/day) : 100

Enter nutrient decay rate (s-1) : 0

Results:

Zone B volume = 230822321.4852 m³

Min, Median, Max Exchange rates = 3.2934e-006 3.3278e-006 3.5293e-006 s-1

Min, Median, Max Flushing Times = 78.7 83.5 84.3 hours

Min, Median, Max ECE = 1.4208e-006 1.5068e-006 1.5225e-006 kg m-3

Min, Median, Max ECE = 0.10148 0.10763 0.10875 umol/l

```
>>
```

Figure 2. Screen dump of the input to and output from the model during a mode 1 'Single Value' application for a site 'test'. OWExR is the name of the model, and V, E and C contain the output values.

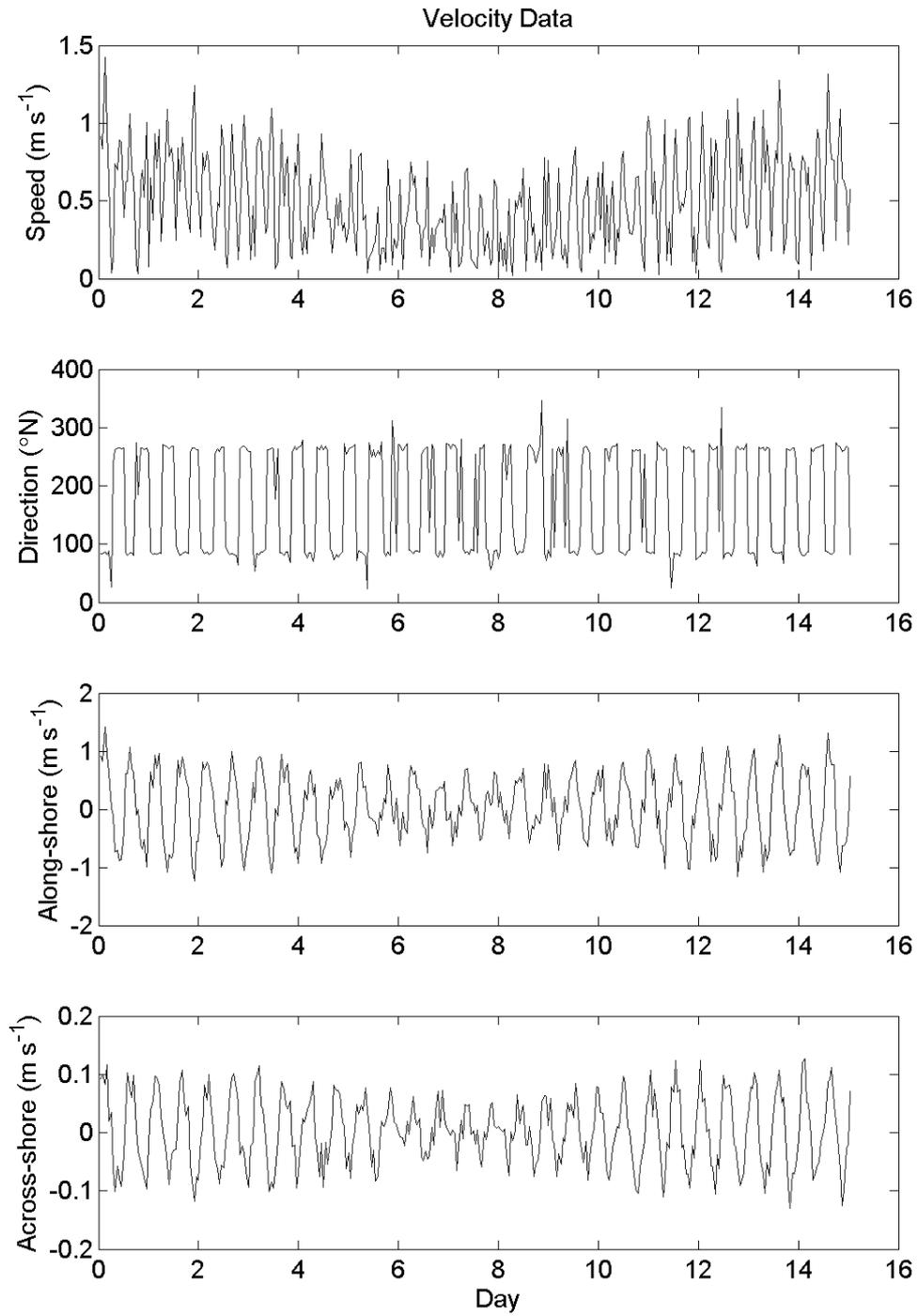


Figure 3. A 15-day time series of current speed and direction, and along-shore and across-shore components of velocity.

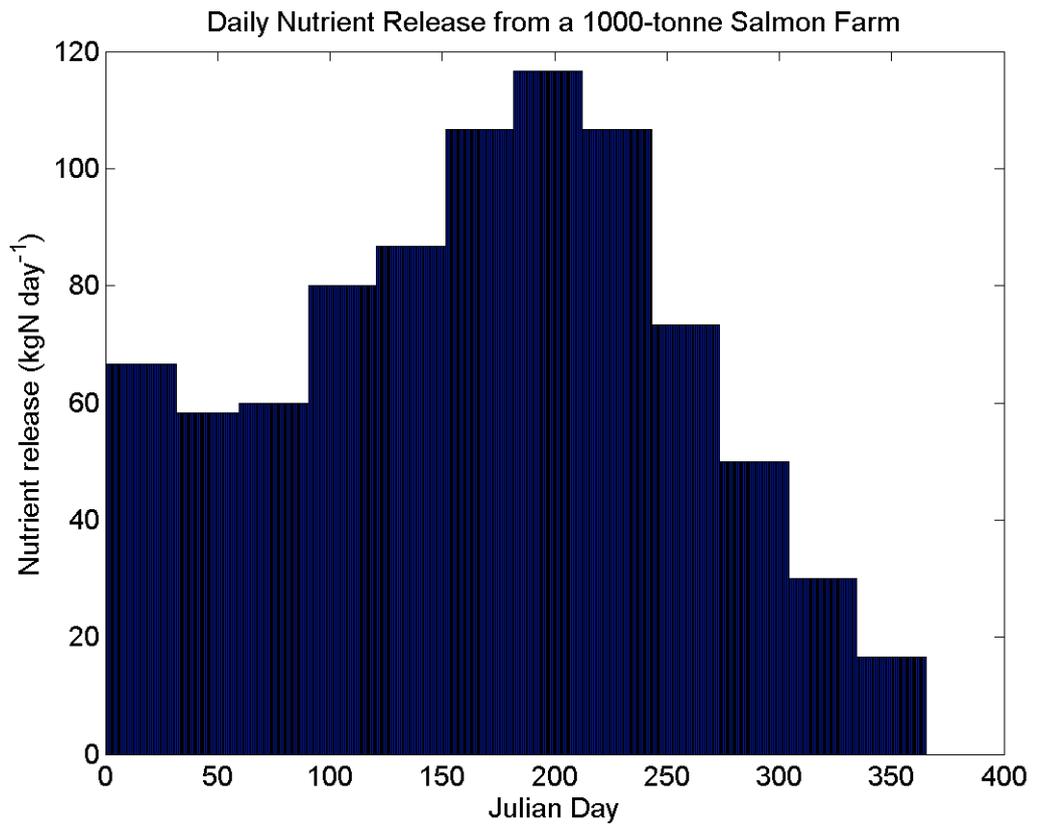


Figure 4. Daily time series of nutrient source (kgN day⁻¹) for a 1000-tonne salmon farm over a calendar year. Values derived from Davies (2000).

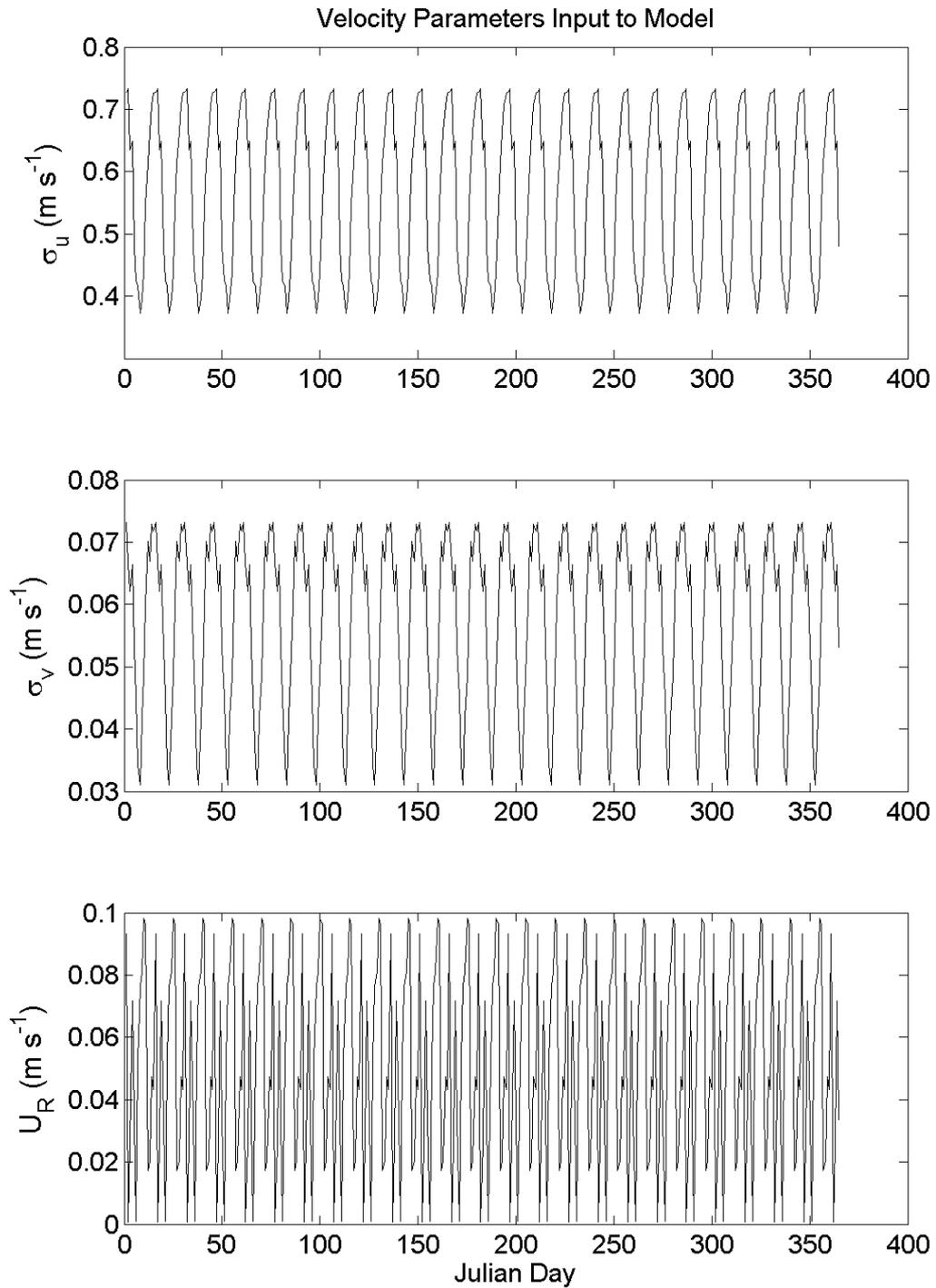


Figure 5. Annual time series of velocity parameters calculated by the model from the velocity time series of Figure 3. The parameters shown are the along-shore standard deviation, σ_u , cross-shore standard deviation, σ_v , and the alongshore residual flow, U_R . The 15-day periodicity resulting from the repeating current meter record is clearly evident in the time series.

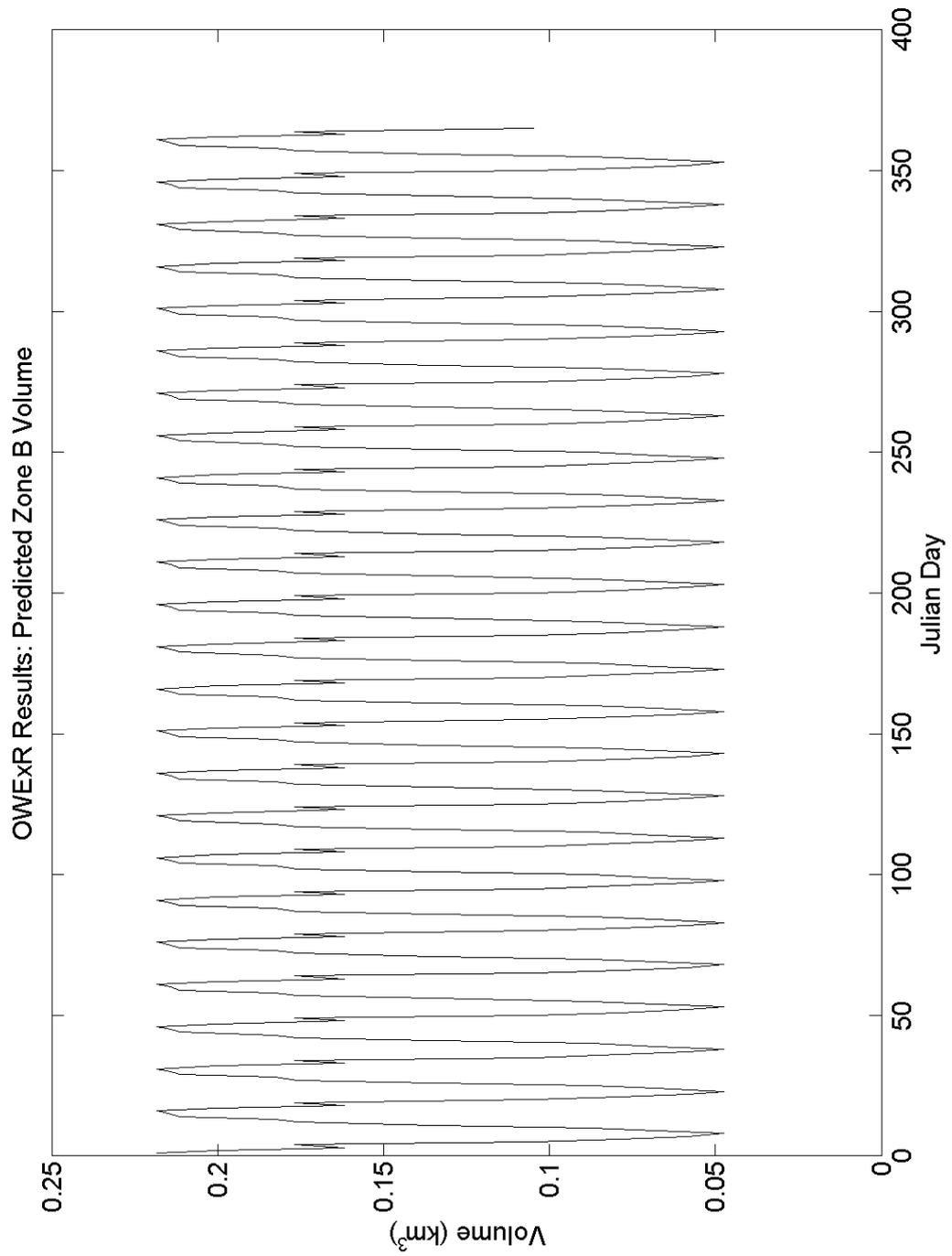


Figure 6. Results from the OWExR model: time series of predicted Zone B volume (km³) for site 'test' calculated from the velocity time series.

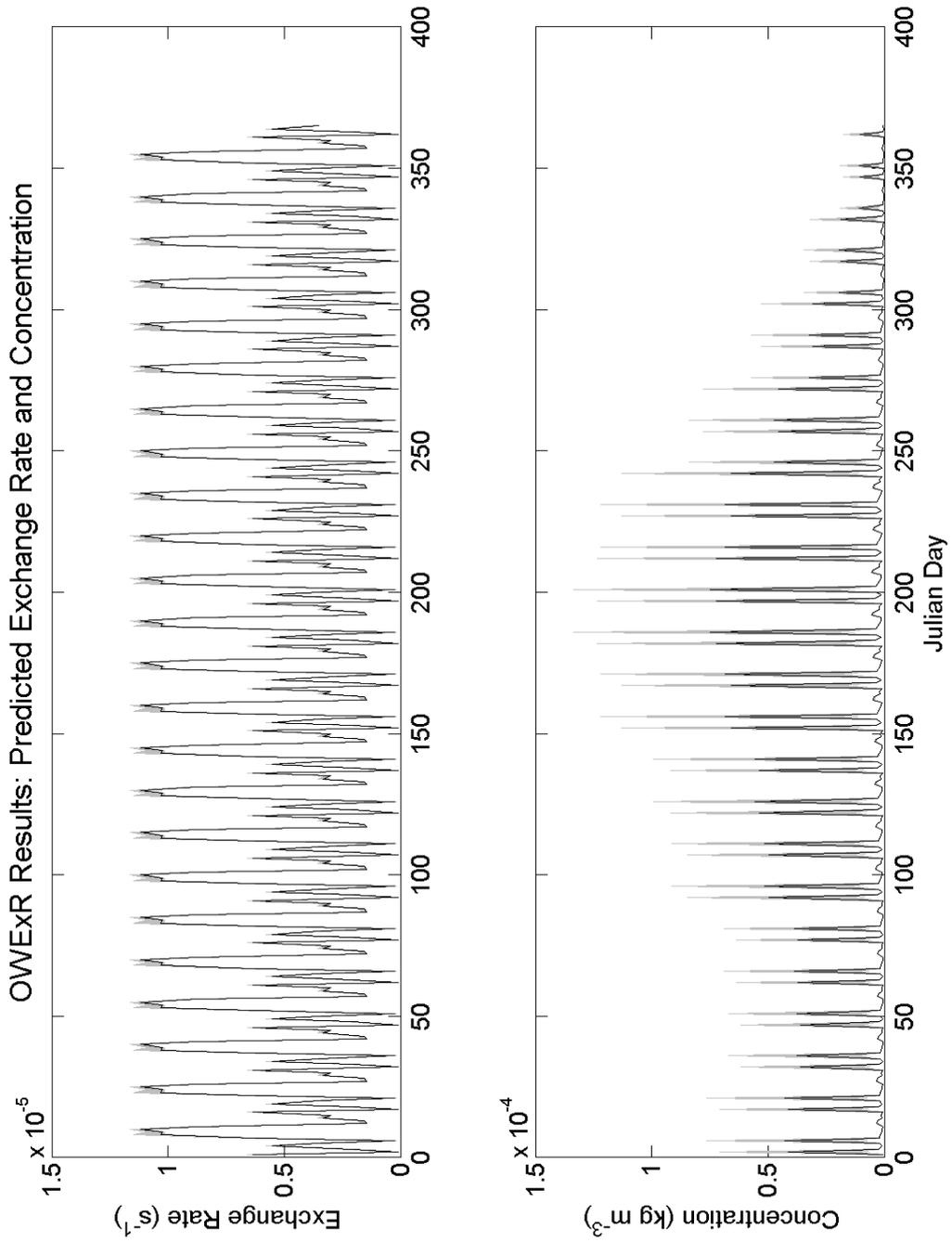


Figure 7. Results from the OWExR model: time series of predicted exchange rate (s⁻¹) and effluent (nutrient) concentration (kg m⁻³). The solid black line indicates the daily median values, the grey shading indicates the range of values from daily minimum to daily maximum.